Riordan Grupları

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ÖZET

$g_0 \neq 0$ ve $f_1 \neq 0$ olmak üzere $g(x) = g_0 + g_1x + g_2x^2 + \cdots$ ve $f(x) = f_1x + f_2x^2 + f_3x^3 + \cdots$ formal kuvvet serileri olsun. j. sütunu $g(x)f(x)^j$, $j = 0,1,2,\ldots$ tarafından üretilen dizinin elemanlarından oluşan matris (g, f) ile gösterilsin. Bu şekilde tanımlı bütün sonsuz boyutlu alt üçgen matrislerin kümesi $\mathcal{R}$, . her $(g, f), (u, v) \in \mathcal{R}$ için $(g, f) * (u, v) = (g(u\mathrm{o}f),\ v\mathrm{o}f)$ şeklinde tanımlanan işlem ile bir gruptur. Grubun birim elemanı $I = (1, x)$ ve (g, f) Riordan çiftinin tersi $(g, f)^{-1} = \left(\frac{1}{g o \bar{f}}, \bar{f}\right)$ şeklindedir.

Riordan grup kavramı ilk olarak 1991 yılında Shapiro, Getu, Woon ve Woodson tarafından tanıtılmıştır [1]. Kombinatoriyel özelliklerin gelişmesinde çok önemli bir yere sahip Amerikalı matematikçi John Riordan atfen bu isim verilmiştir. Renzo Sprugnoli, Riordan grupların cebirsel yapılarını incelemiş [2, 3], Paul Barry özel matrisler kullanarak Pascal matrisleri çeşitli özelliklerini elde etmiştir [4]. Tuğlu ve arkadaşları fibonomiyel katsayılar ile Riordan kavramını birleştirmişlerdir [5].

Bu çalışmada $*_q$ ve $*_{\frac{1}{q}}$ işlemleri yardımı ile q −Riodan kavramı tanımlandı. Bu işlemler yardımı ile elde edilen matrislerin yapıları incelenmiş ve çeşitli özellikleri elde edilmiştir [6].

Anahtar Kelimeler : Riordan grup, Pascal matrisi, Formal kuvvet serileri.

ABSTRACT

Let $g(x) = g_0 + g_1x + g_2x^2 + \cdots$ and $f(x) = f_1x + f_2x^2 + f_3x^3 + \cdots$. be two formal power series with $g_0 \neq 0$ $f_1 \neq 0$. The j-th column of matrix is generated by $g(x)f(x)^j$, for $j = 0,1,2,\ldots$. and the resulting matrix is denoted by (g, f). The Riordan group $\mathcal{R}$ is a set of lower triangular matrices, for all $(g, f), (u, v) \in \mathcal{R}$ the group law is $(g, f) * (u, v) = (g(u\mathrm{o}f),\ v\mathrm{o}f)$. The identity element of this group is $I = (1, x)$ and the inverse of $(g, f)$is $(g, f)^{-1} = \left(\frac{1}{g o \bar{f}}, \bar{f}\right)$

In 1991, The Riordan group, was first introduced by Shapiro, Getu, Woan and Woodson. The Riordan group, named after the combinatorialist John Riordan, was an American mathematician who had a strong influence on the development of combinatorics. Renzo Sprugnoli studied the algebraic structures of the Riordan groups [2, 3], Paul Barry has obtained various properties of Pascal matrices using special matrices [4]. Tuglu et al [5] established a relationship between Riordan arrays and Fibonomial coefficients.

In this study we establish the theory of q-Riordan representation using two new binary operations denoted by $*_q$ and $*_{\frac{1}{q}}$ Then we analyse the structure of the entries in the matrix product of general q-Riordan matrices [6].

Key Words: Riordan group, Pascal matrice, Formal power series

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